# Evaporation from a Water Surface: Theory and Experiment

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Evaporation from a horizontal water surface that is exposed to the natural environment is analysed. An approximate equation is deduced that predicts the turbulent convection mass transfer or evaporation rate for cases where the water surface temperature is measarably higher than that of the ambient air. An empirical equution is recommended in the case where the temperature difference is relatively small and for application at night. Evaporation rates are measured and the results are found to be in good agreement with predicted values.

#### Additional keywords: Mass transfer

### Nomenclature

### Roman

- c Molar concentration of species  $[mol/m<sup>3</sup>]$
- $C_f$  Friction coefficient
- $c_p$  Specific heat [J/kg K]<br>D Diffusion coefficient [
- Diffusion coefficient  $[m^2/s]$
- g Gravitational acceleration  $[m/s^2]$
- k Thermal conductivity [W/m K]<br>  $h$  Heat transfer coefficient [W/m<sup>2</sup>
- Heat transfer coefficient  $[W/m^2 K]$
- *I* Solar irradiation  $[W/m^2]$ <br>*m* Mass flux [kg/sm<sup>2</sup>]
- Mass flux  $[kg/sm<sup>2</sup>]$
- $p$  Pressure [N/m<sup>2</sup>]
- q Heat flux  $[W/m^2]$ <br>T Temperature  $[{}^{\circ}C$
- Temperature  $[°C$  or K]
- $t$  Time [s]
- $v$  Speed [m/s]
- $w$  Humidity ratio [kg vapour/kg dry air]
- z Coordinate

## Dimensionless numbers

- Le Lewis number,  $k/(\rho c_p D)$ <br>Pr Prandtl number,  $\mu c_p/k$
- Prandtl number,  $\mu c_p/k$
- *Ra* Rayleigh number,  $g \delta_{Du}^3(\rho_{avi} \rho_{avo}) \rho_{av} c_p / (k\mu)$
- Sc Schmidt number,  $\mu / (\rho D)$

#### Greek

- $\alpha$  Thermal diffusivity,  $k/(\rho c_p)$ , [m<sup>2</sup>/s] or solar absorptivity
- Concentration or partial density layer thickness, [m]  $\delta$
- Zenith angle,  $[°]$  $\theta$ ,
- Dynamic viscosity, [kg/ms]  $\mu$
- Density,  $\lceil \text{kg/m}^3 \rceil$  $\rho$
- Relative humidity  $\phi$

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#### **Subscripts** a Air

- $av$  Air-vapour mixture
- avi Air-vapour mixture at initial condition
- avo Air-vapour mixture at  $z = 0$
- $c$  Concentration<br> $D$  Mass diffusion
- Mass diffusion
- en Energy
- exp Experimental
- h Horizontal
- i Initial condition
- $m$  Uniform mass flux
- $\alpha$  At  $z=0$
- $q$  Uniform heat flux<br>T Temperature
- Temperature
- $t$  Time
- $u$  Unstable condition
- v Vapour
- w Water or wind
- z Zenith

# 1. lntroduction

In 1802 Dalton's classical paper entitled "Experimental essays on the constitution of mixed gases; on the force of steam or vapour from water and other liquids at different temperatures, both in a Torricellian vacuum and in air; on evaporation and on the expansion of gas by heat", was published. In this paper Dalton stated that the rate of evaporation from a water surface is proportional to the difference in vapour pressure at the surface of the water and that in the surrounding air, and furthermore that the wind speed affects this proportionality<sup>1</sup>. Subsequently numerous researchers investigated the problem of evaporation on the basis of Dalton's model. A recent critical and comprehensive review of many of the empirical equations employed to predict evaporation rates from water surfaces is presented by Sartori<sup>2</sup>. Other references are listed by Bansal and  $Xi<sup>3</sup>$  and Tang et al.<sup>4</sup>. It follows from these publications that there was essentially no further more detailed theoretical modelling of the process of evaporation of water from a horizontal surface into the nafural environment subsequent to Dalton's publication. Much uncertainty exists and significant discrepancies occur between empirical equations that predict rates of evaporation under different conditions.

# 2. Analysis

In the following analysis an approximate equation is deduced that predicts the convective mass transfer or evaporation rate per unit area from a horizontal water surface exposed to the natural environment. Initially the transfer rate due to natural convection only is deduced and the equation is then extended to make provision for windy (forced convection) conditions.

Consider a stationary semi-infinite fluid (binary mixture consisting of air and water vapour) in which the concentration  $c_{vi}$  of the species of interest (water vapour) is initially uniform.

Beginning with the time  $t = 0$ , the concentration at the  $z = 0$ boundary or surface is maintained at a greater level  $c_{\nu}$  as shown in figure  $1(a)$ .

Water vapour will diffuse into the semi-infinite medium to form a concentration boundary layer, the thickness of which increases with time.



Figure 1. Concentration or temperature distribution in semi-infinite medium

The mathematical equation of time dependent diffusion in a binary mixture, expressed in terms of the molar concentration c is as follows:

$$
D\frac{\partial^2 c}{\partial z^2} = \frac{\partial c}{\partial t} \tag{1}
$$

The diffusion flux is driven solely by the concentration gradient strictly in an isothermal and isobaric medium. Nevertheless, equation 1 is a good approximation in many non-isothermal systems, where temperafure differences are relatively small. If changes in Kelvin temperature are small the diffusion coefficient D can be assumed to be constant.

Equation 1 is analogous to the time-dependent equation for heat conduction into a semi-infinite solid body i.e.

$$
\alpha \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \tag{2}
$$

If the temperature of a semi-infinite solid is initially uniform at  $T_i$  and a sudden increase in temperature to  $T_0$  occurs at  $z = 0$ as shown in figure  $1(b)$ , Schneider<sup>5</sup> shows that the temperature gradient at  $z = 0$  is given by

$$
\frac{\partial T}{\partial z} = (T_i - T_o) / (\pi \alpha t)^{1/2}
$$
\n(3)

The corresponding heat flux is

$$
q_T = -k \frac{\partial T}{\partial z} = \frac{k(T_o - T_i)}{(\pi \alpha t)^{1/2}}
$$
(4)

An effective heat transfer coefficient can be expressed in terms of this heat flux i.e.

$$
h_{\tau_i} = q_{\tau} / (T_o - T_i) = k / (\pi c \alpha)^{1/2}
$$
 (5)

Similarly, by solving equation 2 for the case where the semi-infinite solid at an initial uniform temperature  $T_i$  is suddenly exposed to a constant surface heat flux  $q<sub>a</sub>$ , the latter can, according to Holman<sup>6</sup>, be expressed in terms of an effective surface temperature  $T_{oa}$  as

$$
q_q = k(T_{oq} - T_i) / [2(\alpha t / \pi)^{1/2}] \tag{6}
$$

The corresponding effective heat transfer coefficient is defined as

$$
h_{qt} = q_q / (T_{oq} - T_i) = k / [2(\alpha t / \pi)^{1/2}] \tag{7}
$$

It follows from equations 5 and 7 that for the same temperature difference i.e. for

$$
(T_{oq} - T_i) = (T_o - T_i)
$$
  
\n
$$
h_{qt} / h_{Tt} = \pi / 2 = h_q / h_T
$$
\n(8)

Although equation  $2$  is applicable to a solid, it is a good approximation when applied to a thin layer of gas or vapour near a solid surface.

Due to the analogy between mass and heat transfer the solution of equation I gives the following relations corresponding to equations 3 to 8 respectively:

If the initial concentration at  $z = 0$  is suddenly increased to  $c_{\nu o}$ 

$$
\frac{\partial c_{\nu}}{\partial z} = (c_{\nu i} - c_{\nu o})/(\pi D t)^{1/2}
$$
\n(9)

The corresponding vapour mass flux is

$$
m_{\nu} = -D\frac{\partial c}{\partial z} = \left(c_{\nu o} - c_{\nu i}\right) \left[D/(\pi t)\right]^{1/2} \tag{10}
$$

An effective mass transfer coefficient can be expressed in terms of this vapour mass flux i.e.

$$
h_{Dt} = m_v / (c_{vo} - c_{vi}) = [D / (\pi t)]^{1/2}
$$
 (11)

If vapour is generated uniformly at a rate  $m_{vm}$  at  $z = 0$ , this mass flux can be expressed in terms of an effective concentration  $c_{\text{vom}}$  to give analogous to equation 6

$$
m_{vm} = D(c_{vom} - c_{vi})/[2(Dt/\pi)^{1/2}]
$$
  
=  $(c_{vom} - c_{vi})(\pi D/t)^{1/2}/2$  (12)

The corresponding effective mass transfer coefficient is defined as

$$
h_{Dmt} = m_{vm} / (c_{vom} - c_{vi}) = (\pi D / t)^{1/2} / 2
$$
 (13)

It follows from equations 11 and 13 that for the same effective difference in concentration i.e. for

$$
(c_{vom} - c_{vi}) = (c_{vo} - c_{vi})
$$
  
\n
$$
h_{Dmt} / h_{Dt} = \pi / 2 = h_{Dm} / h_{D}
$$
\n(14)

These latter equations are applicable in the region of early developing concentration distribution in a semi-infinite region of air exposed to a water or wet surface. According to Merker<sup>7</sup> for a Rayleigh number  $Ra \ge 1101$ , unstable conditions prevail with the result that water vapour is transported upwards away from the wetted surface by means of "thermals"as shown in figure 2.



Figure 2: Flow development of air adjacent to surface

spatial frequency and rate of production are found to increase The generation of such thermals is periodic in time, and both with an increase in heating rate.

For an analysis ofthe initial developing vapour concentration distribution near the suddenly wetted surface at  $z = 0$ , consider figure  $1(a)$ .

The approximate magnitude of the curvature of the concentration profile is the same as the change in slope  $dc_{v}$  /  $\partial z$  across i.e. the relatively small concentration layer thickness or height  $\delta_p$ 

$$
\frac{\partial^2 c_{\nu}}{\partial z^2} \approx \frac{\left(\partial c_{\nu} / \partial z\right)_{z=\delta_D} - \left(\partial c_{\nu} / \partial z\right)_{z=0}}{\delta_D - 0}
$$
(15)

Figure 1(a) suggests the following concentration gradien scales:

$$
(\partial c_{v}/\partial z)_{z=\delta_{D}} = 0,
$$
  

$$
(\partial c_{v}/\partial z)_{z=0} \approx (c_{vi} - c_{vo})/\delta_{D}
$$

Substitute these gradients into equation 15 and find

$$
\partial^2 c_v / \partial z^2 \approx -\left(c_{vi} - c_{vo}\right) / \delta_D^2 \tag{16}
$$

The approximate magnitude of the term on the right-hand side of equation 1 can be deduced by arguing that the average concentration of the  $\delta_{D}$ -thick region increases from the initial value  $c_{vi}$  by a value of  $(c_{vo} - c_{vi})/2$  during the time interval of length  $t$ .

$$
\partial c_{v} / \partial t = (c_{vo} - c_{vi})/(2t)
$$
 (17)

According to equations 1, 16 and 17 find

$$
-(c_{vi} - c_{vo})/\delta_D^2 \approx (c_{vo} - c_{vi})/(2Dt)
$$
  
or  

$$
\delta_D = (2Dt)^{1/2}
$$

The concentration layer becomes unstable when

$$
Ra = g\delta_{Du}^{3} \left( \rho_{avi} - \rho_{avo} \right) \rho_{av} c_{p} / (k\mu) = 1101
$$
 (19)

where 
$$
\rho_{av} = (\rho_{avo} + \rho_{avi})/2
$$
  
At this condition

$$
\delta_{Du} = 10.33 \left[ k\mu / \left\{ g(\rho_{\text{avi}} - \rho_{\text{avo}}) \rho_{\text{av}} c_p \right\} \right]^{1/3}
$$
 (20)

From equations 18 and 20 find

$$
t_u = 53.31 [k\mu / \{g(\rho_{\text{avi}} - \rho_{\text{avo}})\rho_{\text{av}}c_p\}]^{2/3}/D \tag{21}
$$

Substitute equation 21 into equation 11 to find

$$
h_{Dl}\left[k\mu/\{g(\rho_{\text{avi}}-\rho_{\text{avo}})\rho_{\text{av}}c_p\}\right]^{1/3}/D=0.0773\tag{22}
$$

The average mass transfer coefficient during the period  $t$  is found by integrating equation 11 i.e.

$$
h_D = 2[D/(\pi t_u)]^{1/2} = 2h_{Dt}
$$

or upon substitution of equation 22

$$
h_{Dt} [k\mu/\{g(\rho_{avi} - \rho_{avo})\rho_{av}c_p\}]^{1/3}/D = 0.155
$$
 (23)

If the surface generates vapour at a uniform rate it follows from equation 14 that

$$
h_{Dm} = \pi h_D / 2
$$

or

$$
h_{Dm}[k\mu/\{g(\rho_{avi} - \rho_{avom})\rho_{av}c_p\}]^{1/3}/D = 0.243
$$
 (24)

It is stressed that these equations are only applicable to the first phase of the heat or mass transfer process (growth of concentration layer) and do not include the second phase during which thermals exist (breakdown of concentration layer). No simple analytical approach is possible during this latter phase, although the mean mass transfer coefficient during the breakdown of the concentration layer will probably not differ much from the first phase. This would mean that the mean mass transfer coefficient over the cycle of conduction or concentration layer growth and breakdown is of approximately the same value as that obtained during the first phase of the cycle.

By following a procedure similar to the above, the analogous problem ofheat transfer during natural convection above a heated horizontal surface for a constant surface temperature of  $T<sub>o</sub>$  can be analysed to find according to Kröger<sup>8</sup>

$$
h_r \left[ \mu T / \left\{ g(T_o - T_i) c_p k^2 \rho^2 \right\} \right]^{1/3}
$$
  
=  $h_r \left[ k \mu / \left\{ g(\rho_i - \rho_o) c_p \rho \right\} \right]^{1/3} / k = 0.155$  (25)

and for the case of uniform heat flux  $q$ 

$$
h_q \Big[ \mu T / \Big\{ g \Big( T_{oq} - T_i \Big) c_p k^2 \rho^2 \Big\} \Big]^{1/3}
$$
  
=  $h_q \Big[ k \mu / \Big\{ g \Big( \rho_i - \rho_{oq} \Big) c_p \rho \Big\} \Big]^{1/3} / k = 0.243$   
where  $\rho = (\rho_i + \rho_o) / 2$  (26)

Note the similarity between equations 23, 24, 25 and 26 respectively. These equations are applicable to natural convection mass and heat transfer respectively.

In the absence of winds, effective values of  $\rho_{\alpha}$  in equations 23 and 24 and  $\rho_i$  in equation 25 and 26 respectively, change with time  $(t > t_u)$ .

During windy periods (forced convection) evaporation rates generally increase with increasing wind speed. According to

R & D Journal, 2007, 23 (3) of the South African Institution of Mechanical Engineering 7

(18)

the Reynolds-Colburn analogy and the analogy between mass and heat transfer, the following relations  $exist<sup>6</sup>$ 

$$
\frac{h_w Pr^{2/3}}{\rho c_p v_w} = \frac{C_f}{2} = \frac{h_{Dw} Sc^{2/3}}{v_w}
$$
\nor

\n
$$
h_{Dw} = C_f v_w / (2Sc^{2/3})
$$
\n(27)

In general the rate of mass transfer or evaporation from a horizontal wetted surface at a uniform concentration  $c_{v0}$  is thus

$$
m_{\nu o} = \left[ h_D + h_{Dw} \right] (c_{\nu o} - c_{\nu i}) \tag{28}
$$

Substitute equations 23 and 27 into equation 28 and find

$$
m_{vo} = [0.155 D\{g(\rho_{ovi} - \rho_{avo})\rho_{ov}c_p/(k\mu)\}^{1/3} + C_f v_w/(2Sc^{2/3})]
$$
  
\n
$$
\times (c_{vo} - c_{vi})
$$
  
\n
$$
= [0.155\{g\mu^2(\rho_{avi} - \rho_{avo})c_p/(k\rho_{ov}^2)\}^{1/3}/Sc + C_f v_w/(2Sc^{2/3})]
$$
  
\n
$$
\times (c_{vo} - c_{vi})
$$
\n(29)

For relatively small temperature differences, the concentrations in equation 29 canbe replaced by the partial vapour pressures i.e.  $c_v = p_v / R_v T_{oi}$  where  $T_{oi} = (T_o + T_v)/2$  and  $R_v = 461.52 \text{ J}/\text{Kg K}$ . Furthermore, for air-water vapour mixtures  $S_c \approx 0.6$ .

Substitute these values into equation 29 and find

$$
m_{vo} = 5.6 \times 10^{-4} \Big[ \Big\{ g \mu^2 (\rho_{avi} - \rho_{avo}) c_p / (k \rho_{av}^2) \Big\}^{1/3}
$$
  
+ 2.72  $C_f v_w$   $\Big] (p_{vo} - p_{vi}) / T_{oi}$  (30)

Similarly, if the vapour is generated uniformly at  $z=0$  find the rate of evaporation according to equations 24,27 and 28 i.e.

$$
m_{vom} = 8.78 \times 10^{-4} \left[ \left\{ g \mu^2 (\rho_{avi} - \rho_{avom}) c_p / (k \rho_{av}^2) \right\}^{1/3} + 1.735 C_f v_w \right] (\rho_{vo} - p_{vi}) / T_{oi}
$$
\n(31)

Since the thermal conductivity of water is not negligible, it is not possible to achieve a truly uniform heat flux situation. The value of the dimensionless mass transfer coefficient as given by equation 24 may thus be less than 0.243, i.e. it will be some value between  $0.243$  and  $0.155$  as given by equation 23. Burger and Kröger<sup>9</sup> report the results of experiments conducted during analogous heat transfer tests between a low thermal conductivity horizontal surface and the environment. They obtain a value of 0.2106 instead ofthe theoretical value of 0.243 given in equation 26 and the analogous equation 24. They furthermore obtain a value of  $C_f$  = 0.0052 based on a wind speed measured 1 m above the test surface. With these values equation 26 applied over a weffed surface can be extended to become

$$
h_q \left[ \mu / \{ g c_p \rho_{av} k^2 (\rho_{avi} - \rho_{avom}) \} \right]^{1/3}
$$
  
= 0.2106 + 0.0026 v<sub>w</sub>  $\left[ \rho_{av}^2 / \{ \mu g (\rho_{avi} - \rho_{avom}) \} \right]^{1/3}$  (32)

When density differences are very small and conditions near

the surface are relatively stable or at night when  $T_{eq} < T_i$  and the heat flux is uniform they recommend

$$
h_q = 3.87 + 0.0022 \frac{v_w \rho_w c_p}{(\mu c_p / k)^{2/3}}
$$
 (33)

In cases where  $T_{oq} < T_i$  and  $h_q$  according to equation 33 is larger than the value of  $h_q$  obtained according to equation 32, the former is applicable. If the above values (0.2106 and  $C_f$  = 0.0052) are substituted into equation 31 find

$$
m_{vom} = 7.61 \times 10^{-4} \left[ \left\{ g \mu^2 (\rho_{\text{avi}} - \rho_{\text{avom}}) c_p / (k \rho_{\text{av}}^2) \right\}^{1/3} + 0.0104 v_w \right] (\rho_{vo} - \rho_{vi}) / T_{oi}
$$
(34)

A mass transfer coefficient that is analogous to equation 33 is given by

$$
h_{Dm} = \frac{h_q}{\rho_{av} c_p} \left(\frac{Pr}{Sc}\right)^{2/3} = \frac{3.87}{\rho_{av} c_p Sc} \left(\frac{Pr}{Sc}\right)^{2/3} + \frac{0.0022 v_w}{Sc^{2/3}} \tag{35}
$$

The corresponding uniform mass transfer rate is

$$
m_{\text{vom}} = \left[\frac{3.87}{\rho_{\text{av}}c_p} \left(\frac{Pr}{Sc}\right)^{2/3} + \frac{0.0022v_w}{Sc^{2/3}} \right] \left(\frac{p_{\text{vo}} - p_{\text{vi}}}{R_v T_{\text{oi}}} \right) \tag{36}
$$

For  $Sc \approx 0.6$ ,  $Pr \approx 0.7$  and  $Rv = 461.52$  J/kg K find

$$
m_{\nu_{om}} \approx \left[0.0093/(\rho_{av} c_p) + 6.7 \times 10^{-6} v_w\right] \frac{(p_{\nu_o} - p_{\nu i})}{T_{oi}} \tag{37}
$$

Equation 37 is found to be in good agreement with equations recommended by Tang et al.<sup>4</sup>. This expression is applicable at night and during the day when the value for  $m_{\nu o m}$  is found to be larger that that given by equation 34.

The density of the ambient air is given by

$$
\rho_{\text{avi}} = (1 + w_i) \left[ 1 - w_i / (w_i + 0.622) \right] p_a / (287.08 T_i)
$$
\n(38)

where  $p_a$  is the pressure and  $T_i$  is the temperature of the ambient air.

If the relative humidity of the ambient air is known

$$
\phi = p_{vi} / p_{vsi} \tag{39}
$$

the vapour pressure in the air can be expressed as

$$
p_{vi} = \phi \, p_{vsi} \approx \phi \cdot 2.368745 \times 10^{11} \exp(-5406.1915/T_i)
$$
 (40)

The humidity ratio of the ambient air is given by

$$
w_i = 0.622 p_{vi} / (p_a - p_{vi})
$$
\n(41)

Similarly the density of the saturated air at the surface of the water is

$$
\rho_{\text{avom}} = (1 + w_o) \cdot [1 - w_o / (w_o + 0.622)] p_a / (287.08 T_o)
$$
 (42)

where

$$
w_0 = 0.622 \ p_{vo} / (p_a - p_{vo}) \tag{43}
$$

R & D Journal, 2007, 23 (3) of the South African Institution of Mechanical Engineering

and

$$
p_{vo} \approx 2.368745 \times 10^{11} \exp(-5406.1915/T_o) \tag{44}
$$

Sartori<sup>2</sup> lists many empirical correlations that predict the rate of evaporation. The values given by these equations may differ significantly over a range as shown by the shaded area in figure 3. In part this may be due to errors in the determination of the water surface temperafure, as well as different vapour concentration distributions above water surfaces having different areas. In figure 3 the rate of evaporation is shown as a function of wetted surface temperature  $T<sub>a</sub>$  and ambient air temperatures  $T_1 = T_0 - 5$ , a relative humidity of  $\phi = 45$  per cent and a wind speed of  $v_{\rm w} = 3$  m/s at about 1m above the wetted surface. Sartori<sup>2</sup> recommends the equations proposed by the  $WMO^{10}$  and  $McMillan<sup>11</sup>$  as shown in figure 3. Equation 34 is also shown in figure 3 for an ambient pressure of  $p = 10^5$  N/m<sup>2</sup>.

#### 3. Experimental Apparatus and Procedure

To experimentally determine the rate of evaporation from a water surface exposed to the natural environment, the use of an evaporation pan as shown in figure 4 was employed. It consisted of a 50 mm thick horizontal polystyrene plate having an effective upper surface area of approximately  $0.97 \text{ m}^2$ , which was painted with a waterproof matt black paint. A 3 mm high bead of silicon sealant was run along the perimeter of the pan, with the purpose of containing a I - 2 mm deep layer of water. Five type-T thermocouples were embedded flush in the surface of the plate with the purpose of measuring the water temperature. Four of the five were positioned in the corners of the pan, 150 mm from adjacent sides, while the fifth was placed in the centre. The pan was surrounded by a dry stony surface.



Figure 3: Rate of evaporation. Shaded area shows range of results and correlations of various researchers according to Sartori<sup>2</sup>.

The wind speed, ambient air- and dew-point temperatures were measured with the aid of a weather station at a height of 1 m above the ground<sup>8</sup>. A Kipp and Zonen pyranometer was used to measure the total incident solar radiation on the surface,



Figure 4: Evaporation pan

while diffuse solar radiation readings were measured by shielding the pyranometer from direct sunlight for a period long enough for stable measurements to be taken. A11 tests were conducted on clear sunny days.

The evaporation rate from the water surface was measured by adding consecutive quantities of water (500 ml) to the evaporation pan at atemperafure similar to that of the remaining water, and recording the period of time taken for each to evaporate. With the average temperature of the water known during the particular period, the mass flowrate or evaporation rate could be determined. This evaporation rate will be denoted by  $m_{\text{env}}$ .

By applying an energybalance to the surface ofafilm ofwater on an insulated base that is exposed to the natural environment as shown schematically in figure 4, it is possible to obtain an expression for the rate of evaporation per unit surface area i.e.

$$
m_{en} = [I_h \alpha_w - \varepsilon_w \sigma (T_{oq}^4 - T_{sky}^4) - h_q (T_{oq} - T_i)]/i_{fg}
$$
(45)

where  $I_h \alpha_w$  represents the solar radiation absorbed by the surface,  $\varepsilon_w \sigma (T_{og}^4 - T_{sky}^4)$  is the sky radiation while  $h_q(T_{og} - T_i)$ is the convective heat transfer rate. The absorptivity of solar radiation at the surface of the water is given by Holman<sup>6</sup> and can be approximated by the following equation

$$
\alpha_w = 0.989 - 2.05/(90 - \theta_z) \tag{46}
$$

where  $\theta_z$  is the zenith angle measured in degrees.

The surface emits radiation to a sky temperature  $T_{sky}$ , which can be calculated according to

$$
T_{sky} = \varepsilon_{sky}^{1/4} \cdot T_a \tag{47}
$$

Berdahl and Fromberg<sup>12</sup> express the emissivity of the sky  $\varepsilon_{sk}$ 

(48)

during the day as

$$
\varepsilon_{sky} = 0.727 + 0.0060 T_{dp}
$$

while at night

$$
\varepsilon_{sky} = 0.741 + 0.0062 T_{dp} \tag{49}
$$

where  $T_{dp}$  is the dew-point temperature measured in degrees Celsius.

The heat transfer coefficient  $h_q$  is given by equation 32 while equation 33 is applicable when  $T_{oa} > T_i$  or when the value of  $h_a$  according to equation 33 during the day is larger than the value obtained according to equation 32. The heat transfer through the polystyrene plate is negligible.

According to Holman<sup>6</sup> the intensity of solar radiation in clear water at a distance z from the surface is given by

$$
I_z = 0.6 I_h e^{-0.16z} \tag{50}
$$

For a thin film of thickness  $z \approx 0.0015$  m the heat flux near the surface of the film is thus approximately

$$
k\frac{dT}{dz} \approx I_z \approx 0.6 I_h \tag{51}
$$

or the surface temperafure can be described by

$$
T_{og} \approx T_{o\ measured} - 0.6 \frac{I_h z}{k} \approx T_{o\ measured} - 0.0015 I_h
$$
 (52)

An analysis was performed on equation 34 to determine the sensitivity of the expression to an error of  $\pm 1$ <sup>o</sup>C in measured water temperature. It was found that equation 34 is very sensitive to water temperature early in the morning and towards evening, while a difference in the predicted evaporation rate of approximately l0% was found during the day. These findings suggest that extreme care should be taken when measuring the water surface temperature  $T_{oa}$ .

Tests were conducted on the 13<sup>th</sup> and 14<sup>th</sup> of April 2005 at the University of Stellenbosch Solar Energy Laboratory (33.93" S,







Figure 6: Measured temperatures

18.85" E), at an altitude of 100 m above sea level with an ambient pressure of 100990 Pa. The associated weather data is given in figures 5 and 6 and the results are shown in figures 7 and 8.

Figure 7 shows a comparison between the experimental and theoretical evaporation rates between approximately 8 h and l5 h (14'n April). Note that no wind was present until roughly l0 h; figure 7 shows almost 'sfunted' evaporation rates until this point which may be due to the accumulation of moist air above the water surface, which lead to an increase in  $\rho_{vi}$  and thus reduced evaporation. Figure 8 shows evaporation rates over a period of 24 hours. Note that a negative evaporation rate, or condensation in the form of dew is found to occur during the night when the water surface temperature  $T_{oq}$  is less than the dew-point temperature  $T_{dp}$ .

Table I compares the experimentally measured quantity of water evaporated between 8.319 h and 14.752 h, with the values predicted by equations 34 and 45. The error margin is calculated with respect to the experimentally measured quantity.

Results from above show that equation34 predicts the measured evaporation rate most accurately, while equation 45 is also within reasonable accuracy.

Both equations predict condensation onthe water surface during night-time operation, while the magnitudes of the predicted rates seem to be of the same order.



Table 1: Comparison between cumulative evaporation rates

#### 4. Conclusion

Horizontal natural water or wetted surfaces (surface area of approximately one square metre) having a relatively low thermal conductivity and exposed to solar radiation, transfer both mass and heat to the nafural environment. Under these conditions the rate of evaporation at the wetted surface will be determined primarily by the heat flux due to solar radiation and can thus be evaluated according to equation 34 for cases where  $\rho_{\text{avi}}$  $> \rho_{\text{avo}}$ , the free stream vapour concentration  $c_{vi}$  is

10 R & D Journal, 2007, 23 (3) of the South African Institution of Mechanical Engineering



Figure 7: Comparison between evaporation rates



Figure 8. Diurnal evaporation rates

uniform and the wind speed is less than 4 m/s at a height of <sup>1</sup> m above the surface<sup>8</sup>. Although equation 34 is an approximation it is based on a sound theoretical approach. Equation 37 is recommended for use during the night-time and when it gives a value of  $m_{\text{v}om}$  that is larger than that given by equation 34.

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